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Comment on “Once more about the $K\bar{K}$ molecule approach to the light scalars”

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In this manuscript we reply to the criticisms of our paper [Eur. Phys. J. A 24, 437 (2005)] raised in a recent preprint by Achasov and Kiselev [hep-ph/0606268] and demonstrate that all their criticisms are completely irrelevant and misleading.

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In a recent paper [1] we considered the radiative decay $\phi \rightarrow \gamma a_0/f_0$ in the molecular ($K\bar{K}$) model of the scalar mesons ($a_0(980)$, $f_0(980)$). In particular, we showed that there was no considerable suppression of the decay amplitude due to the molecular nature of the scalar mesons. In addition, as a more general result we demonstrated that, as soon as the vertex function of the scalar meson is treated properly, the corresponding decay rates become very similar to those for pointlike (quarkonia) scalar mesons, provided reasonable values are chosen for the range of the interaction. We also confirmed the range of order of $10^{-3} \div 10^{-4}$ for the branching ratio obtained in Refs. [2, 3, 4] within the molecular model.

Our results imply that differences in the concrete calculations of the rate for $\phi \rightarrow \gamma a_0/f_0$ for a molecular and a quark ($q\bar{q}$ or $qq\bar{q}\bar{q}$) structure of the light scalar mesons can be only hidden in the effective coupling constants. In either case — regardless whether the scalars are molecules or not — the algebra how to calculate the rates is identical. Furthermore, the form factors representing the involved ($K\bar{K}$) vertices should be similar. In fact, we have argued that the effective couplings extracted from the ϕ radiative decays seem to be more consistent with those expected for a molecular state or, at least, for a state containing a large admixture of the $K\bar{K}$ molecule. This is not surprising since for the kaon loop mechanism of

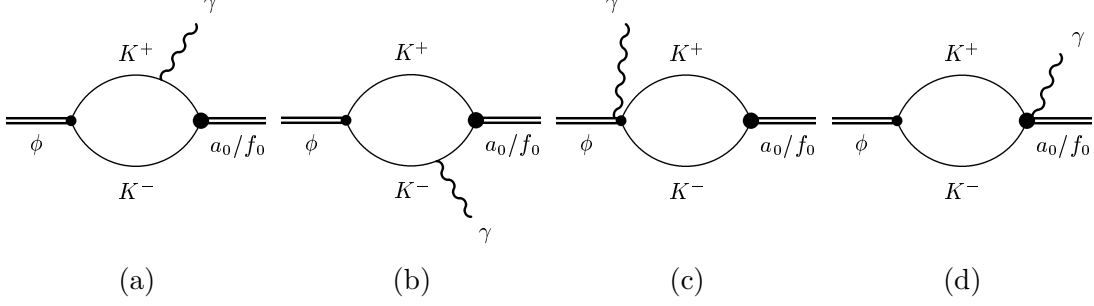


FIG. 1: Diagrams contributing to the amplitude of the radiative decay $\phi \rightarrow \gamma a_0/f_0$.

radiative decay to be operative, one needs the scalar effective coupling to be sizeable, which, in turn, requires a sizeable admixture of the $K\bar{K}$ in the scalar wavefunction, as demonstrated in [5]. In other words, it means that the scalar radiative decay in the kaon loop mechanism proceeds via the molecular component of the wavefunction.

As a reaction to our work a critical comment appeared [6], where the authors dispute our results and, specifically, where they claim that our paper [1] is “misleading”. We shall refute the arguments of the authors of Ref. [6] below in detail one by one.

But first, let us briefly recall the essentials of the formalism. If the scalar meson is considered to be point-like, the current describing the transition $\phi \rightarrow \gamma S$ ($S = a_0$ or f_0) is given by the diagrams (a)-(c) of Fig. 1, and can be written as

$$M_\nu = \frac{eg_\phi g_S}{2\pi^2 i m_K^2} I(a, b) [\varepsilon_\nu(p \cdot q) - p_\nu(q \cdot \varepsilon)], \quad (1)$$

where p and q are the momenta of the ϕ meson and the photon, respectively, m_K is the kaon mass, g_ϕ and g_S are the $\phi K^+ K^-$ and $S K^+ K^-$ coupling constants, ε_ν is the polarisation four-vector of the ϕ meson, $a = \frac{m_\phi^2}{m_K^2}$, and $b = \frac{m_S^2}{m_K^2}$ (we deal here with stable scalars — see the comment on nonstable scalars below). The amplitude (1) is transverse, $M_\nu q_\nu = 0$, and it is proportional to the photon momentum. The form (1) is well known, and the details can be found, for example, in Refs. [7, 8, 9, 10, 11]. Specifically, the total matrix element is finite if evaluated consistently with gauge invariance and the integral $I(a, b)$ entering Eq. (1) remains finite in the limit $a \rightarrow b$.

Now, if the scalar S is treated as a non-point-like object, a corresponding form factor is to be attached to the $K^+ K^- S$ vertex. Note that the appearance of non-point-like vertices is in full agreement with the requirements of Quantum Field Theory (QFT). It can be derived naturally as a solution to the Bethe-Salpeter equation in any suitable QFT-inspired model

for the interaction of kaons which is able to bind a $K\bar{K}$ molecule. Thus it does not, by any means, stem from “... wrong interpretation of the result of calculation in the point like case...”, as was claimed in Ref. [6]. Also an extra graph (graph (d) of Fig. 1) should be added, which accounts for the additional flow of the charge. The form for this extra contact vertex was suggested in Ref. [11], and is based on the minimal substitution considerations. Indeed, let the scalar vertex Γ depend on the total momentum of the scalar P and on the momentum k of the K^+ . Then, making the replacement $k_\mu \rightarrow k_\mu - eA_\mu$, one obtains the “interaction current”

$$-e \frac{\partial \Gamma(k, P)}{\partial k_\nu}. \quad (2)$$

On the other hand, the authors of the preprint [6] repeatedly claim that gauge invariance of the matrix element should be imposed by hand, namely by a subtraction at the point $q = 0$. For example, in the paper [12], devoted to the calculation of the $\phi \rightarrow \gamma S$ width in the molecular model, it is written explicitly: “...the decay amplitude

$$T(p, q) = M(p, q) - M(p, 0),$$

$$M(p, q) = M_1(p, q) + M_2(p, q) + M_3(p, q), \quad (3)$$

where p and q are the four-momenta of the ϕ meson and photon respectively. ... The amplitudes M_1 , M_2 and M_3 correspond to the diagrams Fig. 1(a)-(c), respectively. ...”

Let us start by stressing that, generally speaking, both prescriptions (2) and (3) fail to restore gauge invariance of the amplitude. Indeed, the sum of the graphs depicted at Fig. 1(a)-(c) is given by $M_\nu = eg_\phi g_S \varepsilon_\mu J_{\mu\nu}$ with

$$\begin{aligned} J_{\mu\nu} = & \int \frac{d^4 k}{(2\pi)^4} (2k + 2q - p)_\mu (2k + q)_\nu \Gamma(k, p - q) S(k) S(k - p + q) S(k + q) \\ & + \int \frac{d^4 k}{(2\pi)^4} (2k - p)_\mu (2k - 2p + q)_\nu \Gamma(k, p - q) S(k) S(k - p + q) S(k - p) \\ & - 2g_{\mu\nu} \int \frac{d^4 k}{(2\pi)^4} \Gamma(k, p - q) S(k) S(k - p + q), \end{aligned} \quad (4)$$

where

$$S(k) = \frac{1}{k^2 - m^2 + i0} \quad (5)$$

is the propagator of the K meson, where here and in the following we use the notation $m = m_K$ for simplicity. Gauge invariance requires that the amplitude is transverse, while

$$q_\nu J_{\mu\nu} = \int \frac{d^4 k}{(2\pi)^4} S(k) S(k - p) (2k - p)_\mu [\Gamma(k + q, p - q) - \Gamma(k, p - q)] \neq 0, \quad (6)$$

and both prescriptions (2) and (3) are obviously unable to repair this.

But also here the solution to this problem is well-known in the literature (see, Ref. [13], where we list a few of the papers devoted to this subject). In principle, one is to take into account the strong interaction of the $K\bar{K}$ pair in the intermediate state, i.e. with the interaction kernel responsible for the bound scalar state formation. Inserting the photon line inside these graphs (the so-called structure emission) restores gauge invariance. However, in the soft-photon limit, one can safely do without such complications. Indeed, note that for soft photons (which is obviously the case for the radiative decay under consideration), one may safely use the current (2), so that, *in the prescription (2)*, the piece that violates gauge invariance in Eq. (6) is cancelled by an extra contribution to $J_{\mu\nu}$,

$$- \int \frac{d^4k}{(2\pi)^4} S(k)S(k-p)(2k-p)_\mu \frac{\partial \Gamma(k,p)}{\partial k_\nu}, \quad (7)$$

coming from the contact vertex (2) (through the contact graph depicted in Fig. 1(d)). Therefore, *in the prescription (2)*, the total current *is* transverse [14] in the soft-photon limit, i.e. up to corrections of the order ω/m with ω being the photon energy.

Consider now the prescription (3). Gauge invariance indeed requires the amplitude with neutral particles in the initial and final state to vanish at $q = 0$. However, this alone *does not at all suffice to ensure gauge invariance*, as seen from (6). In addition such a subtraction procedure *is not unique*. The subtracted piece is only defined up to q -dependent contributions that vanish in the limit $q = 0$, but which might spoil the transversality of the transition current. Thus, there are corrections in the order of ω/m too. And, finally, the properly calculated amplitude *must vanish at $q = 0$ by itself, without extra subtraction, finite or otherwise*¹. Thus, based on those considerations one can also put into question the entire subtraction procedure suggested and advocated by one of the authors of the preprint [6]. However, for the present discussion it is important to note that, in the soft photon limit, the subtraction prescription works and, as a matter of facts, *it provides results identical to those given by the prescription (2)*. To show this, let us blindly use the prescription (3).

¹The case of light-by-light scattering mentioned in [6] cannot be considered as a counter-example: the amplitude calculated in the dimensional regularisation scheme or in the background field method nicely vanishes automatically in the limit $\omega \rightarrow 0$.

The expression (4) for the transition current reads at $q = 0$:

$$\begin{aligned}
J_{\mu\nu}(q = 0) = & \int \frac{d^4k}{(2\pi)^4} (2k - p)_\mu (2k)_\nu \Gamma(k, p) S(k) S(k - p) S(k) \\
& + \int \frac{d^4k}{(2\pi)^4} (2k - p)_\mu (2k - 2p)_\nu \Gamma(k, p) S(k) S(k - p) S(k - p) \\
& - 2g_{\mu\nu} \int \frac{d^4k}{(2\pi)^4} \Gamma(k, p) S(k) S(k - p).
\end{aligned} \tag{8}$$

If the Ward identity,

$$2k_\nu S(k) S(k) = -\frac{\partial S(k)}{\partial k_\nu}, \tag{9}$$

is used in the first term above and an integration by parts is performed then the result reads

$$J_{\mu\nu}(q = 0) = - \int \frac{d^4k}{(2\pi)^4} S(k) S(k - p) (2k - p)_\mu \frac{\partial \Gamma(k, p)}{\partial k_\nu}, \tag{10}$$

which is obviously identical to the expression (7) found for the other prescription. Therefore, we conclude that the criticism made in Ref. [6] regarding the alleged violation of gauge invariance by the amplitude derived with the help of the prescription (2) is rather questionable.

After this lengthy introduction let us discuss the main formula of our paper [1]. It is argued there (and confirmed by actual calculations) that, since the amplitude is finite even for the point-like limit, the range of convergence of the integrals involved *is defined only by the kinematics of the problem*. In particular, if both masses, i.e. that of the vector and of the scalar mesons, are close to the $K\bar{K}$ threshold, the integrals converge at $k_0 \sim m$ and for nonrelativistic values of the three-dimensional loop momentum \vec{k} , $|\vec{k}| \ll m$. This allows us to perform a nonrelativistic reduction of the amplitude in the rest-frame of the ϕ meson. The contact scalar vertex is taken in the form (7), yielding for the individual graphs of Fig. 1:

$$\begin{aligned}
J_{ik}^{(a)} = J_{ik}^{(b)} &= -\frac{i}{m^3} \int \frac{d^3k}{(2\pi)^3} \frac{k_i(\vec{k} - \frac{1}{2}\vec{q})_k \Gamma(|\vec{k} - \vec{q}/2|)}{[E_V - \frac{k^2}{m} + i0][E_S - \frac{(\vec{k} - \vec{q}/2)^2}{m} + i0]}, \\
J_{ik}^{(c)} &= -\frac{i}{2m^2} \delta_{ik} \int \frac{d^3k}{(2\pi)^3} \frac{\Gamma(k)}{E_S - \frac{k^2}{m} + i0}, \\
J_{ik}^{(d)} &= -\frac{i}{2m^2} \int \frac{d^3k}{(2\pi)^3} \frac{k_i k_k}{E_V - \frac{k^2}{m} + i0} \frac{1}{k} \frac{\partial \Gamma(k)}{\partial k},
\end{aligned} \tag{11}$$

where $E_V = m_V - 2m$, $E_S = m_S - 2m$. Integration by parts was performed for the last integral, yielding

$$J_{ik}^{(d)} = \frac{i}{2m^2} \delta_{ik} \int \frac{d^3k}{(2\pi)^3} \frac{\Gamma(k)}{E_V - \frac{k^2}{m} + i0} + \frac{i}{3m^3} \delta_{ik} \int \frac{d^3k}{(2\pi)^3} \frac{k^2 \Gamma(k)}{(E_V - \frac{k^2}{m} + i0)^2} . \quad (12)$$

Surprisingly, it is this procedure of integration by parts which caused the main criticism in Ref. [6]. Indeed, it is difficult to understand why the authors of Ref. [6] overlooked the simple fact that, by virtue of Eq. (10), even with the subtraction procedure (3) one would arrive *at the very same result* (12). Schematically, this looks as follows (for the sake of brevity we omit indices). Let us start with the form (11) and denote the sum of the real parts of the integrals $J^{(a)} + J^{(b)} + J^{(c)}$ in Eq. (11) as $J(q)$. Let us follow for a moment the authors of Ref. [6] and introduce the cut-off k_0 to compute the integrals $J(q, k_0)$ and $J^{(d)}(k_0)$. But it is important to realize that such a sharp cut-off obviously breaks gauge invariance – something which the authors of the preprint [6] have either overlooked or tacitly ignored. Then, as a consequence, the sum $J(q, k_0) + J^{(d)}(k_0)$ clearly exhibits a drastic dependence on the cut-off: the limiting value $J(q, \infty) + J^{(d)}(\infty)$ is not reproduced with the values of k_0 up to several GeV (see Figs. 2, 3 of Ref. [6]). This behaviour is demonstrated in Ref. [6] and it is advertised as being the collapse of the approach suggested in Ref. [1]. However, one can easily see that the sum $J(q, \infty) + J^{(d)}(\infty)$ vanishes at $q = 0$, while the sum $J(q, k_0) + J^{(d)}(k_0)$ does not if $J^{(d)}$ is taken in the form (11). Thus, as already mentioned before, such a cut-off imposed by brute force *does not respect gauge invariance* and consequently, and in line with the arguments of one of the authors of the preprint [6], calls for the subtraction:

$$J^R(q, k_0) = J(q, k_0) + J^{(d)}(k_0) - [J(q, k_0) + J^{(d)}(k_0)]|_{q=0} = J(q, k_0) - J(0, k_0). \quad (13)$$

If then $J^{(d)}$ is chosen as defined in Eq. (12) one obtains $J(0, k_0) = J^{(d)}(k_0)$ and, thus, the procedure of integration by parts after performing nonrelativistic reduction is *perfectly justified* even in the subtraction scheme (3) advocated by one of the authors of the preprint [6]. The regularised integral $J^R(q, k_0)$ *converges rapidly and for low virtual momenta of the kaons* – which follows both from our findings and from the one of the preprint [6] (see Fig. 4 of Ref. [6]). In this context, let us note that the authors of Ref. [6] have deliberately forgotten to mention that, for realistic values for the range of the interaction, β , i.e. $\beta \sim 0.6$ GeV or larger, the result for the matrix element is actually very close to the one of the point-like theory.

Therefore, we conclude that there are no flaws in the derivation presented in Ref. [1]. This is in a drastic contrast to the statement of Ref. [6], where it is claimed that high kaon virtualities are dominating the radiative transition amplitude in the point-like limit. Note that, if this would be indeed the case, then the kaon loop mechanism for the ϕ radiative decay must be *completely irrelevant*: there are no infinitely compact states in nature. Besides, the convergence of an integral for values of the loop momentum much larger than all the natural scales present in the problem looks rather suspicious from a physical point of view (and is also aesthetically unacceptable).

A further claim by the authors of Ref. [6] is that “... there is another sloppily build place in Ref. [1]. The authors of Ref. [1] calculate $A(\phi(p \rightarrow \gamma S(p'))$ at $q = 0$, $p^2 = m_\phi^2$ and $(p')^2 = m_S^2$ ($m_S - 2m < 0$), that is, at $p \neq p' + q$...” (see p. 7 (reference [6]) of Ref. [6]). This is simply *not true* and it remains completely unclear to us what made them to draw that conclusion. The amplitude was calculated retaining the full dependence on \vec{q} in Eq. (11). It turned out, however, that, due to the inequality

$$m(E_V - E_S) \gg \vec{q}^2 \approx (E_V - E_S)^2, \quad (14)$$

one may safely omit *numerically* the terms containing \vec{q} in the expression (11). This does not mean that the calculations were performed under the assumption $p \neq p' + q$.²

Yet another wrong statement of Ref. [6] concerns the problem of unstable scalars. It remains unclear how it is related to the “question of principle” raised at p. 7 of Ref. [6], but the actual situation is as follows. An attentive and unbiased reader would have noticed that, while in the main body of our paper [1] the case of the stable scalar with $m_S - 2m < 0$ is considered, the Appendix is entirely devoted to the case of the unstable scalar which can decay into a pair of light pseudoscalars $P_1 P_2$. Contrary to the claims of Ref. [6], the $P_1 P_2$ invariant mass distribution is calculated, both in the differential form and also integrated over the near-threshold mass region, $900 \text{ MeV} < M < m_\phi$. The results presented in [1] appear to be *robust against the inclusion of the finite width of the scalar*.

To summarise, *it is the statement of the preprint [6] to be completely misleading*. There are no reasons at all to believe that the radiative decays of the ϕ meson into scalars would

²Strictly speaking, it is not legitimate to keep subleading terms in q , as both prescriptions (2) and (3) maintain gauge invariance only in the soft photon limit, as was discussed above.

allow to discriminate between molecules and compact states.

Finally, let us reiterate the main results Ref. [1]. There, we have shown that there is no appreciable suppression of the $\phi \rightarrow \gamma S$ width in the molecular model for the scalar mesons, provided reasonable values are chosen for the range of the interaction, $\beta \sim 0.6 - 0.8$ GeV. In the molecular model this range is defined by the mass of the lightest meson participating in the exchange force, that is by the ρ meson. In recent kaon-loop calculations [2, 3, 4] where the scalars are considered as dynamically generated states, that is, as molecules, a range of the interaction of similar order was used, and a good description of the data on the ϕ radiative transitions was achieved. If the scalars are treated as $q\bar{q}$ states, the range of interaction can be estimated from the range of the vertex responsible for the strong decays of the mesons. This can be deduced, for example, within the 3P_0 model for pair creation [15] to be $\beta \sim 0.8$ GeV. Therefore, it is not possible to distinguish between a molecular or a $q\bar{q}$ nature of the $a_0(980)$ and $f_0(980)$ from data on ϕ radiative decays based on the value of the relevant interaction range because they are essentially the same. The only real difference in the concrete calculations of the rate for $\phi \rightarrow \gamma a_0/f_0$ for a molecular and a quark ($q\bar{q}$ or $qq\bar{q}\bar{q}$) structure of the light scalar mesons is hidden in the effective coupling constants. The “compactness” advocated in Ref. [6] as a criterion for the discrimination is completely irrelevant for the ϕ radiative decay.

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